A Common-Period Four-Satellite Continuous Global Coverage Constellation

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This paper describes a new four-satellite elliptic orbit constellation giving continuous line-of-sight coverage of every point on the Earth's surface. It represents a major improvement over the only other known four-satellite continuous global coverage array by reducing the constellation operational altitudes by about one-half and by utilizing a common period for all of the satellites employed.

Nomenclature

A	= semimajor axis of satellite orbit, in units of the Earth's equatorial radius r
a	= semimajor axis, n.mi.
e	= eccentricity
h	= satellite altitude, n.mi.
J_2	= coefficient of the second harmonic of the Earth's potential function
i	= inclination, deg
M	= mean anomaly, deg
P	= semiparameter, $= a(1-e^2)$
p	= perpendicular distance from Earth center to satellite plane, n.mi.
r	= satellite radius, n.mi.
R_e	= mean radius of Earth, = 3442 n.mi.
Δr_{max}	= radial perturbation, ft
S_1, S_2, S_3, S_4	= satellite designator
S_1', S_2', S_3', S_4'	= satellite surborbital points
T	= constellation period, h
σ	= satellite visibility angle ("look angle") above horizon, deg
ω	= argument of perigee, deg
$\dot{\omega}$	= rotation of perigee, deg/yr
Ω	= right ascension of ascending node, deg
$\dot{\Omega}$	= regression of line of nodes, deg/yr
μ	= Earth's gravitational constant, $= 1.4077 \times 10^{16}$ ft^3/s^2

Subscripts

a	= apogee
d	= disturbing body
e	= Earth
l	= lunar
p	= perigee
s	= solar

Background

MOST satellite constellation studies addressing the issue of Earth coverage have concentrated on circular orbits.[1-8] Studies prior to 1965 concentrated quite naturally on constellations in low Earth orbit, since neither the booster capability for placing satellites into geostationary orbits nor the sensor capability for operating effectively at such ranges had yet been developed.[1,2] Constellations with elliptic orbits have been considered and utilized for some applications, primarily missions where a coverage bias favoring the Northern Hemisphere is desired.[9,10]

The continued popularity of the circular-orbit geostationary communications satellites has led to an overcrowding in this prime neighborhood of space "real estate" (i.e., 19,323 n. mi. altitudes in or near the equatorial plane). These "synch-eq" orbits are attractive for most applications, since they allow the use of fixed Earth antennae and result in constant distances from the satellite to any given point of latitude. Three or four such satellites suitably spaced do, in fact, constitute a constellation that provides line-of-sight coverage to most of the Earth's surface. Obviously, the portions not covered are those at and near the North and South Poles. These polar regions are becoming increasingly important. The need for global satellite services such as communications, navigation, and weather information dictates that future satellite constellations provide continuous global coverage. Furthermore, the provision of this coverage with a minimum number of satellites has important economic ramifications. When one considers satellites costing in excess of $50 or $100 million each and satellite booster rockets having comparable fabrication and launch costs, the advantage of providing continuous global coverage with one or two fewer satellites per constellation is significant.

Several authors have addressed the specific question, "What is the minimum number of satellites required to ensure continuous Earth coverage?" Two earlier studies concluded that this minimum number was six. Throughout the 1970's, it was generally thought that the minimum number of satellites required to give continuous global coverage (including the polar regions) was five. A credible proof of this minimum number has been presented by Ballard,[6] but it is valid only for circular-orbit constellations.

In 1984, the author presented the concept of a four-satellite elliptical-orbit constellation giving continuous global coverage. It was based on a three-satellite elliptic-orbit continuous hemispherical coverage constellation of 78 h or longer, with a fourth circuit orbit satellite having a period one-half that of the other three. Although this constellation provided continuous global coverage with four satellites, the long period of the three elliptic orbits and the lack of symmetry with the fourth satellite member of the constellation resulted in undesirable design and operational differences.

Introduction

The new four-satellite constellation described in this paper uses common period orbits and can maintain continuous global coverage at approximately one-half the altitude of the earlier four-satellite constellation. Since the orbital periods, inclinations, and eccentricities of the satellites in the new constellation are identical, most of the perturbations will be equal and will not affect the relative phasing and orientation within

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the constellations. Overall advantages of this new satellite constellation are commonality of design, identical ΔV requirements, and lower gains required (due to lower operating altitude).

Several coverage theorems and corollaries (developed by the author) apply to both hemispheric and global coverage.[10] Only theorem I and corollary II are needed to define the conditions sufficient for global coverage.

Theorem I: If a plane containing three satellites does not intersect the Earth or if this plane is tangent to the Earth at some point, then every point within the spherical triangle on the Earth's surface formed by the satellites' suborbital points is visible from at least one of the satellites. (See Fig. 1.)

Corollary II: If the Earth is completely enclosed within a tetrahedron formed by four planes (each containing three satellites), then any point on the Earth's surface is visible from at least one of the satellites, by successive use of theorem I. (See Fig. 2.)

To obtain complete, continuous global coverage, it is sufficient to show that the planes of the tetrahedron always encompass the Earth, without ever intersecting it, as the tetrahedron changes shape or warps during the constellation period. The planes can momentarily become tangent to the Earth's surface, while the visibility criteria are still met. This leads to the use of the descriptive term "osculating planes." The tangent condition will occur for some critical minimum constellation period, where the planes momentarily "kiss" the Earth's surface and then rise to higher altitude. If the perpendicular distances to the planes are less than the Earth's radius, then the planes must intersect the Earth. If the perpendicular distances of the planes remain always greater than the Earth's radius, then the planes do not intersect the Earth's surface.

In this paper, a spherical Earth is assumed. Since we are concerned with coverage of the entire globe, it does not matter whether we introduce Earth rotation or not. Later, we shall consider a "pseudo-Earth" or "pseudoplanet" with a period equal to the satellite period, as a means of visualizing the constellation mechanics.

Constellation Description

The orbital parameters (ephemerides) of a four-satellite continuous coverage constellation are given in Table 1. This constellation was derived from the four-sided regular polyhedron, or platonic solid, known as the tetrahedron. The tetrahedron is assumed to be initially placed with two edges in horizontal planes. It is then perturbed, or flattened, such that the angles of inclination from the horizontal are decreased. The satellite orbits are placed parallel to the planes of this perturbed tetrahedron, passing through the Earth's center. Each of the satellite orbits is then made elliptic, with the eccentricity lying in a suitable range. The ellipses are so arranged that two of the satellites have their perigees in the Northern Hemisphere ($\omega = +90$ deg), while the other two have their perigees in the Southern Hemisphere ($\omega = -90$ deg). Mean anomalies for the starting positions of the satellite orbits are then selected so that the first satellite pair has one satellite S_1 at perigee and the other S_3 at apogee. The other two satellites are placed midway (in time) between apogee and perigee (i.e., S_4 at $M=90$ deg and S_2 at $M=270$ deg). An isometric drawing of a typical configuration is shown in Fig. 3. (The bolder portions of the orbital traces represent those segments nearer the viewer.) A breakdown of the constellations into two pairs is shown in Fig. 4. Since the ephemerides in Table 1 are given in inertial coordinates, one can use these values for any constellation or satellite period one wishes to select. One can thus vary the period, above some absolute lower limit set by satellite plane intersection with the Earth's surface or atmosphere. This absolute lower limit on the constellation period for complete and continuous satellite visibility has been found by numerical calculations to be 26.49 h, after optimizing the inclination and eccentricity combination of orbital parameters. The values of arguments of perigee and right ascensions of

Table 1 Orbital parameters[a]

Satellite	i	e	ω	Ω	M
1	31.3	0.263	−90	0	0
2	31.3	0.263	+90	90	270
3	31.3	0.263	−90	180	180
4	31.3	0.263	+90	270	90

[a] T_c must be equal to or greater than 26.49 h to ensure continuous global coverage.

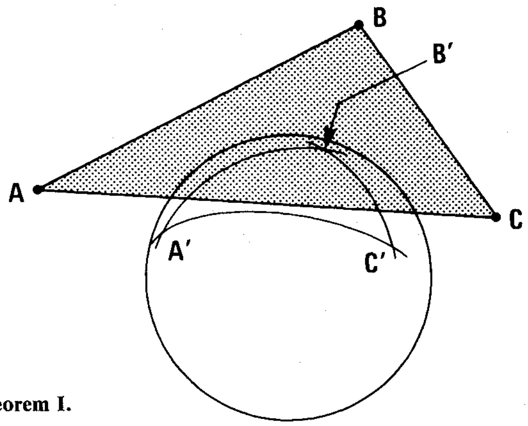

Fig. 1 Theorem I.

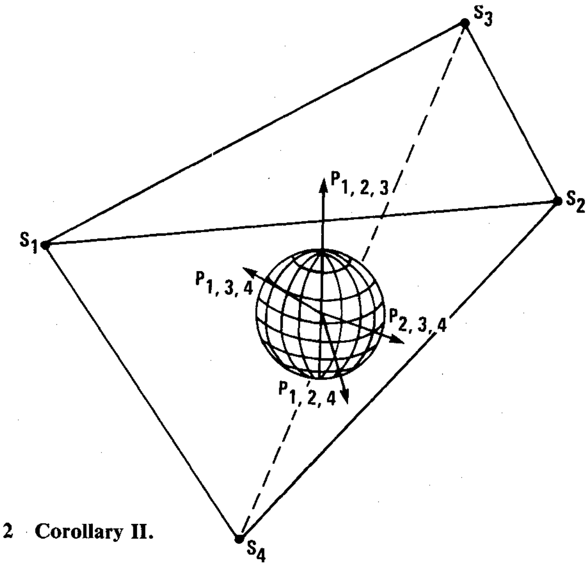

Fig. 2 Corollary II.

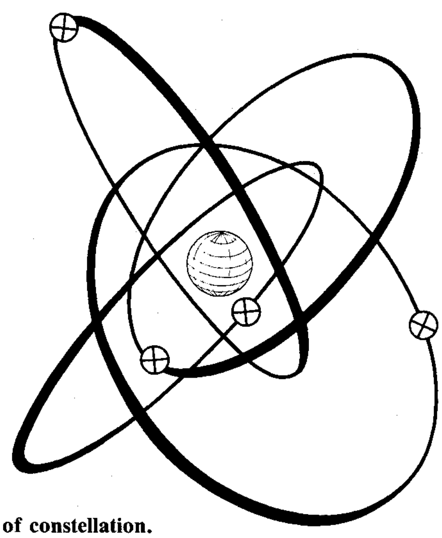

Fig. 3 Isometric of constellation.

ascending nodes remain fixed, since varying these two parameters experimentally always led to a degradation in constellation coverage.

For all periods in excess of 26.49 h, there will be a range of orbital parameters (such as inclination, eccentricity, arguments of perigee, etc.) over which the continuous global coverage can be maintained. As the satellite periods (and thus altitudes) increase, these allowable variations of orbital parameters will widen; but it will still be necessary to maintain the perigees of two satellites in the Northern Hemisphere and the perigees of the two others in the Southern Hemisphere in order to ensure the desired result of continuous global coverage.

Figure 5 shows the minimum visibility angle (angle the satellite appears above the horizon) obtainable as a function of constellation period. It should be noted that this minimum visibility angle (or "look" angle) occurs only periodically in time and then only in a limited geographical region. Most of the time, over the surface of the Earth, much larger visibility angles are the rule.

For comparison with synchronous (24 h) constellations having a semimajor axis of 22,767 n.mi., apogee and perigee radii and altitudes are presented in Table 2 for constellations having greater periods. The eccentricity in all cases is 0.263. Apogee and perigee altitudes given for a hypothetical four-satellite 24 h array would not result in a continuous coverage constellation. It should be noted that the 48 h example constellation has a perigee height of 23,193 n.mi., only 3868 n.mi. above circular synchronous altitude. The apogee height for the 48 h constellation is 42,203 n.mi., which is less than twice synchronous altitude. It is well known that the synchronous equatorial belt is becoming extremely crowded with satellites. The higher, continuous coverage constellations of this paper represent one possible solution to this problem, as they are at higher altitudes and in more highly inclined orbits.

Perpendicular Distance to Plane

Using a computer model that generates elliptical orbit parameters and then generates the equation of the planes passing through three satellites at a time, the absolute value of the perpendicular distance p from Earth's center to the satellite planes may be obtained. This perpendicular distance is shown for one plane of the four satellite continuous coverage model, for $T=27$ and 48 h in Fig. 6. The other three planes will be similar, but displaced by 90 deg phase angles (since the satellites are symmetrical in quarter periods).

It should be noted that as the period of the constellation with given ephemerides is increased, the minimum value of the length of the perpendicular also increases. For a period of 26.49 h and for the optimal values, $i=31.3$ deg and $e=0.263$, each of the planes will become tangent to the Earth's surface twice per orbital (or constellation) period. For longer constellation periods and for the same optimal inclination and eccentricity, the minimum separation distances between the planes and the Earth's surface will increase. This will translate into a greater value of σ, or "look angle," as shown in Fig. 5.

Minimum Look-Angle Contours

Figure 7 plots the relationship of the inclination angle, eccentricity, and minimum look angle. Because of the shape of these contours, they have become known as "banana charts." The plot shown is for a typical 48 h tetrahedral constellation. This period is particularly interesting since it has repeating ground tracks, is high enough above the critical 26.49 h period to ensure positive minimum look angles, and results in quasistationary geographical coverage patterns. These coverage patterns may be shifted in longitude quite easily by selection of the orientation of the constellation's ascending nodes. Some reshaping in latitudinal coverage is also possible, through adjustments to constellation orbital inclination and eccentricity. An interesting possibility is the use of a 48 h constellation with orbital inclinations of 28.5 deg to correspond

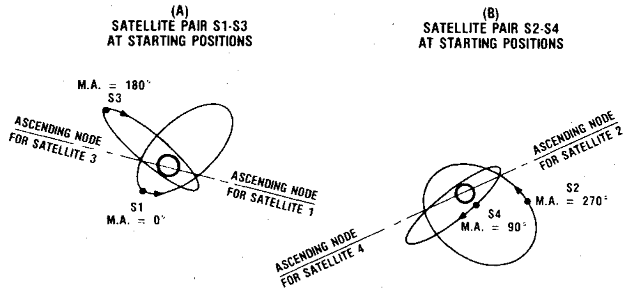

Fig. 4 Satellite starting positions.

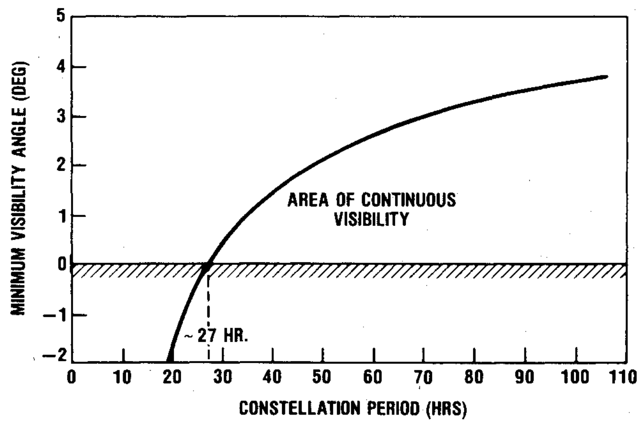

Fig. 5 Minimum visibility angle vs constellation period.

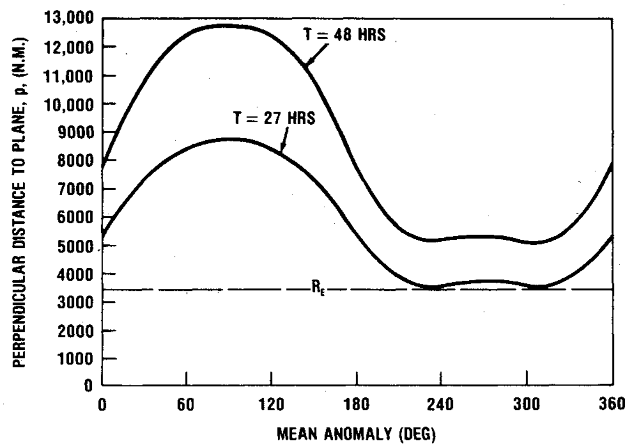

Fig. 6 Perpendicular distance from Earth center to a typical constellation plane.

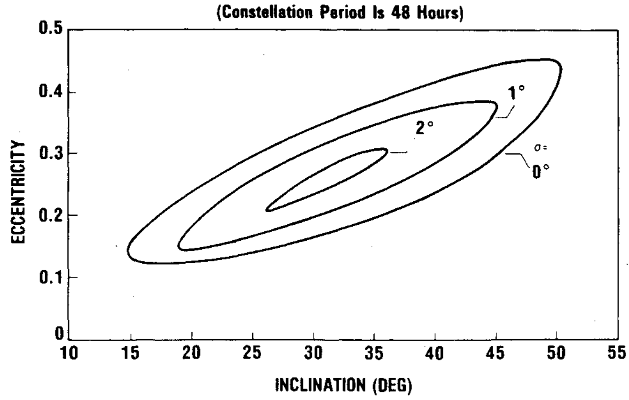

Fig. 7 Visibility angle contour plot.

with the minimum energy launch from Kennedy Space Flight Center. The eccentricity that would best suit this inclination is 0.23, from Fig. 7. For this combination, the minimum look angle will still be over 2 deg, according to Fig. 7.

Explanation of the Constellation

The combined effect of planetary rotation and the movement of a synchronous inclined elliptic orbit can result in a nearly circular ground track.[11] Further, the movement of the ground track will be either clockwise or counterclockwise for arguments of perigee of either +90 or −90 deg, respectively.

To gain a complete understanding of this constellation, it is useful to consider a "pseudoplanet" rotating with the same period as the constellation. Then, the ground tracks of the Southern Hemisphere perigee satellites, (S_1 and S_3) will be almost circular and will coincide for the proper combination of inclination and eccentricity. The track will be counterclockwise when viewed from a point in space. Similarly, the other satellite pair (S_2 and S_4) will rotate clockwise since their perigees are both in the Northern Hemisphere. If one views both pairs of satellites from a single point in space through their apparent axis of rotation in the equatorial plane, the satellites will appear in a roughly cruciform pattern around this axis. Since all four satellites are now apparently rotating in the same direction, the integrity of the tetrahedron is preserved. (See Fig. 8.) Great circle arcs drawn on the surface of the "pseudoplanet" at successive points in time through the first pair of satellites go through an apparent rotation. For continuous coverage, one satellite of the opposite pair must always remain on one side of each great circle arc, with the other satellite remaining on the other side. Stating it another way, the tetrahedron will warp and stretch, but it will not collapse and will not contact the planet's surface. Thus, the planet will be continuously and completely covered by corollary II. Confirmation of this characteristic may be seen in Fig. 9, which was computer generated. It shows a continuous coverage constellation at successive points in time throughout a complete period. Ten-degree increments of mean anomaly M are used. The corners of the tetrahedron represent the satellite positions. At $M_1=0$, the starting position for the constellation, the lower left satellite S_1 is at perigee, while the upper left satellite S_3 is at apogee. Satellites S_2 and S_4 appear superimposed; since $M_2=270$ deg and $M_4=90$ deg and their orbit perigees are in the Northern Hemisphere they are located in the Southern Hemisphere. For the $M_1=0$ snapshot, the left-hand line represents a tetrahedron edge and the two right-hand lines are both planes viewed edge-on. The tetrahedron can be seen to rotate clockwise and warp as M is incremented throughout the period.

Since the minimum satellite period providing continuous global coverage is 26.49 h, a true Earth synchronous (24 h period) constellation is ruled out. This is rather unfortunate, as it would permit the use of two satellites with nearly circular ground tracks centered, say, over the Western Hemisphere at 100°W longitude. The other pair would have its ground tracks at 80°E longitude over Eurasia.

However, if a 48 h period is selected for the satellite constellation, a repeating ground track is obtained for each satellite, as shown in Fig. 10. This figure also shows the satellite starting positions (suborbital points) listed in Table 1.

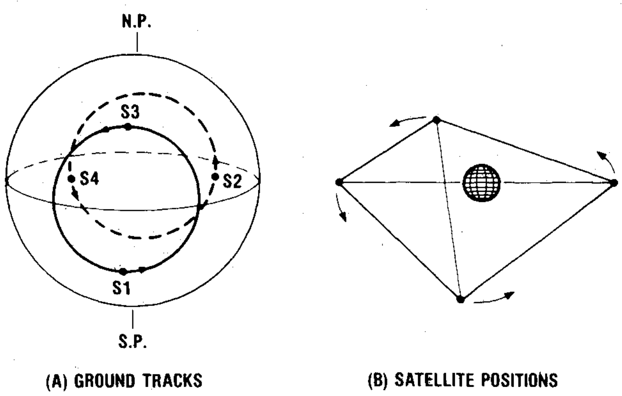

Fig. 8 Constellation explanation, using "pseudoplanet" reference frame.

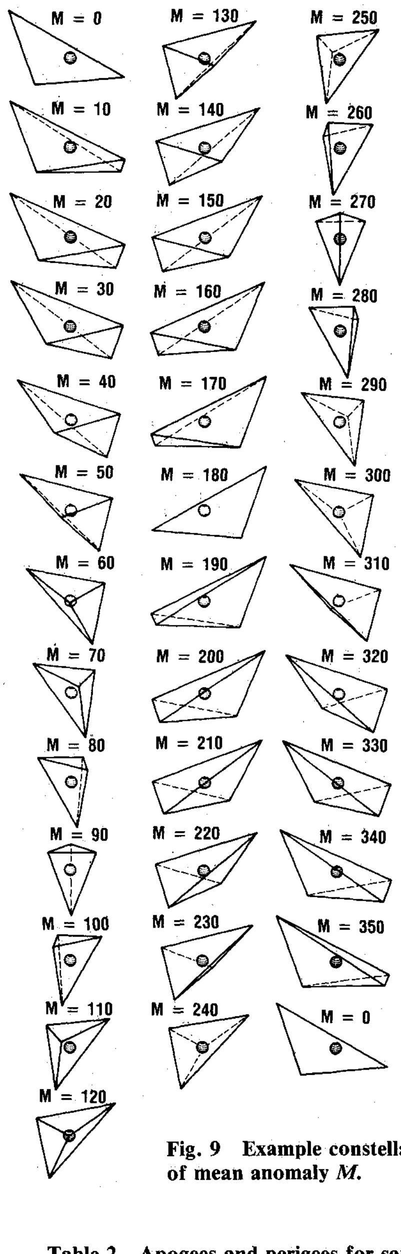

Fig. 9 Example constellation at 10 deg increments of mean anomaly M.

Table 2 Apogees and perigees for satellite array ($e=0.263$)

T, h	a, n. mi.	r_a, n.mi.	h_a, n.mi.	r_p, n.mi.	h_p, n.mi.
24.0[a]-
22767	28755	25313	16779	13337
26.49	24316	30711	27269	17921	14479
30.0	26419	33367	29925	19471	16029
36.0	29833	37679	34237	21987	18545
48.0	36140	45645	42203	26635	23193
60.0	41937	52966	49524	30908	27466
72.0	47357	59812	56370	34902	31460
84.0	52483	66286	62844	38680	35238
96.0	57369	72457	69015	42280	38839
120.0	66571	84079	80637	49063	45621

[a]Not a continuous coverage option.

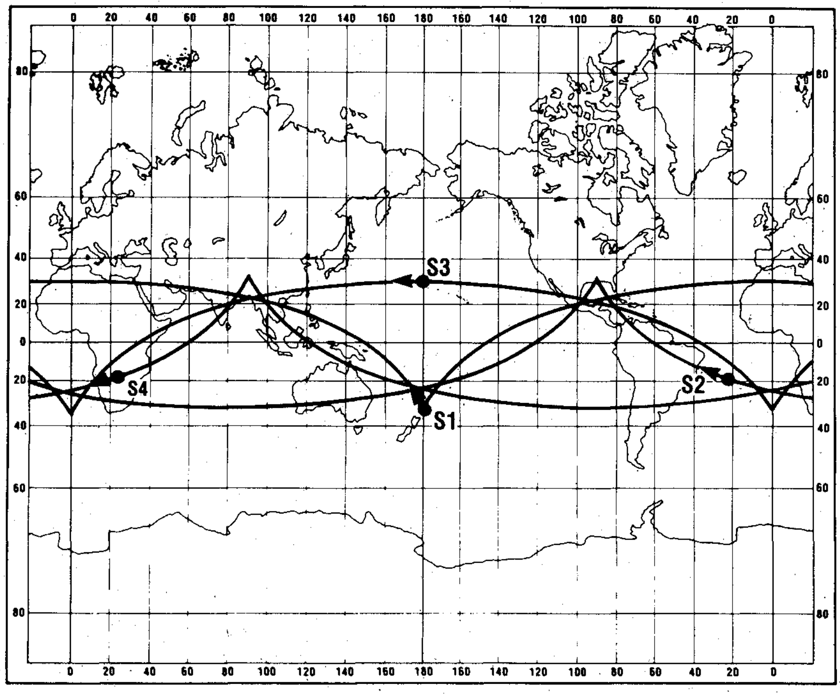

Fig. 10 Example continuous coverage constellation satellite ground track ($T=48$ h, $i=33$ deg, $e=0.28$).

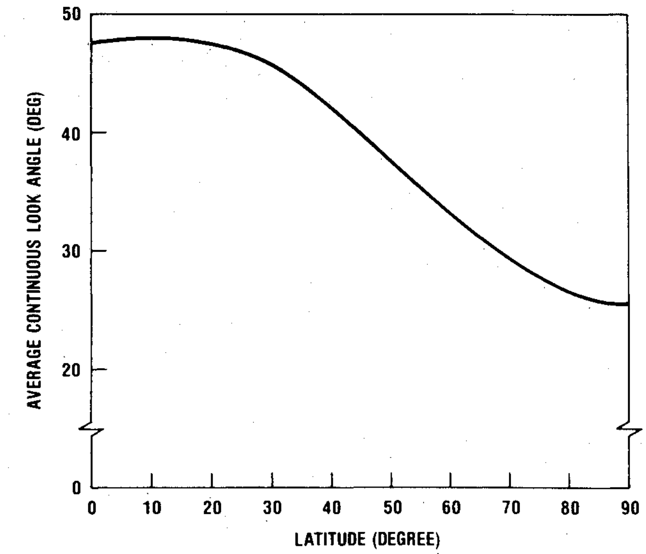

Fig. 11 Example constellation mean look angle as a function of latitude.

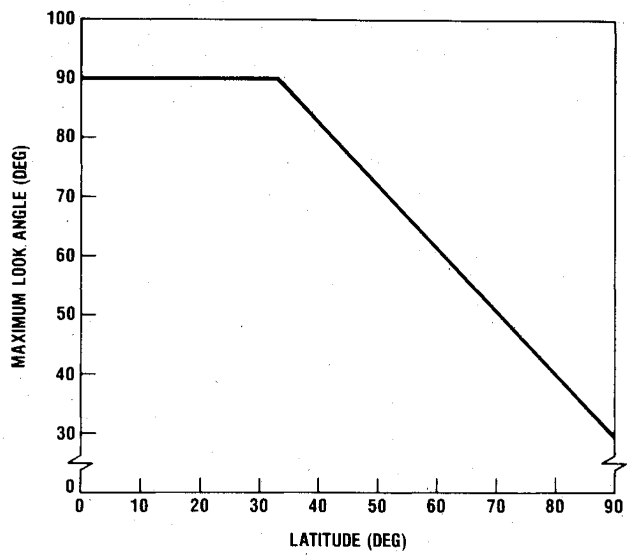

Fig. 12 Example constellation maximum look angle as a function of latitude.

Note that the perigee points are cusped on the ground trace and represent periods when the satellites appear almost stationary to an observer on Earth. The apogee points are marked by a rapid motion past the meridians of longitude, resulting in the grounds tracks having a very small amount of curvature. Since the orbits are higher than synchronous altitude, the satellites appear to pass an observer on Earth from east to west.

Latitudinal Coverage Statistics

A computer model was used to generate the statistics of coverage as a function of latitude for the constellation. For a 48 h period, continuous coverage constellation, with $i=33$ deg, $e=0.28$, the mean, maximum, and minimum look angles are presented as a function of observer latitude, in Figs. 11-13.

Geographic Coverage Charts

As evident in Fig. 10, the 48 h constellation has repeating ground tracks that lead to a distinct geographic pattern for the mean, minimum, and maximum look angles. The satellite system designer can select noncritical geographic locations in which to locate these look-angle minima.

Figures 14-16 show the coverage pattern for mean, minimum, and maximum look angles for the example constellation (which has $i=33$ deg and $e=0.28$). A higher inclination angle and greater eccentricity can be used, say $i=40$ deg and $e=0.34$, which is a point along the axis of Fig. 7. The Earth coverage for this version of the constellation should, intuitively, give improved coverage of the polar regions and somewhat reduced coverage of the equatorial regions of Earth. Figure 17 shows the mean look-angle contours for this case and does, in fact, show improved polar area coverage (while maintaining continuous visibility).

If periods are selected to yield nonrepeating ground tracks, the longitudinal coverage statistics tend to blur into constant values for each latitude line.

Effects of Perturbations

The question of whether the continuous coverage constellations described herein will continue to function properly over the satellite lifetimes in the face of perturbing forces must be addressed. Accordingly, an investigation was made into which disturbing forces affect the orbital parameters and overall constellation integrity.

Obviously, perturbations due to atmospheric drag may be dismissed due to the high altitudes of the satellites.

The effects of Earth oblateness should be considered next; these effects are predominant in the region from a few hundred miles orbital altitude out to 24 h orbital altitudes. There, solar and lunar attractions are about equally important.[12] In the supersynchronous region of most interest, where the continuous coverage constellations will operate (i.e., T greater than 26.5 h), lunar and solar attractions become the major sources of perturbation.

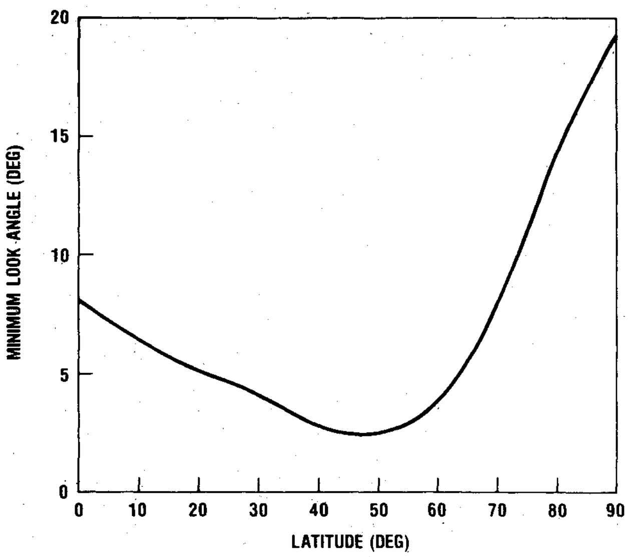

Fig. 13 Example constellation minimum look angle as a function of latitude.

Both lunar and solar perturbations result primarily in a nodal regression of the vehicle orbital plane about the normal to the orbit plane of the perturbing body.[12] Since the orbital parameters of each of the four satellites are the same, the constellation satellites should individually and collectively regress equally. Therefore, the satellite integrity as to its ability to maintain continuous coverage should not be degraded by the lunar or solar perturbations. Also, the amount of regression is small; it was calculated for the 48 h continuous coverage array using

$$\dot{\Omega}_d = -\frac{3}{8}\frac{\mu_d}{r_d^3}\sqrt{\frac{a^3}{\mu}}\frac{\cos i_d}{(1-e_d^2)^{3/2}}\times\cos i\left(\frac{2+3e^2}{\sqrt{1-e^2}}\right) \tag{1}$$

from Ref. 12. The resultant regressions were found to be

$$\dot{\Omega}_\ell = -3.513 \text{ deg/yr} \tag{2}$$

$$\dot{\Omega}_s = -1.633 \text{ deg/yr} \tag{3}$$

Thus, it appears that the lunar attraction accounts for slightly over two-thirds of the total regression rate from third-body forces, with the balance due to solar attraction.

In addition, there is also a radial or tidal perturbation,

$$\Delta r_{\max} = \frac{\mu_d}{\mu}\cdot\frac{r_c^4}{r_d^3} \tag{4}$$

for circular orbits. This approximation to the moderate eccentricity should suffice to determine roughly the magnitude of perturbation. For the 48 h example constellation, the total radial perturbation due to the combined attraction of the sun and the moon is of the order of magnitude of 2000 ft.

The two major perturbations due to Earth oblateness are rotation of the perigee and regression of the line of nodes. The former may require a periodic orbital adjustment or stationkeeping maneuver to keep the perigees at or near $\omega = \pm 90$ deg. The latter perturbation is not a major concern, as all of the satellites in the constellation will regress equally; thus, the constellation integrity will be maintained.

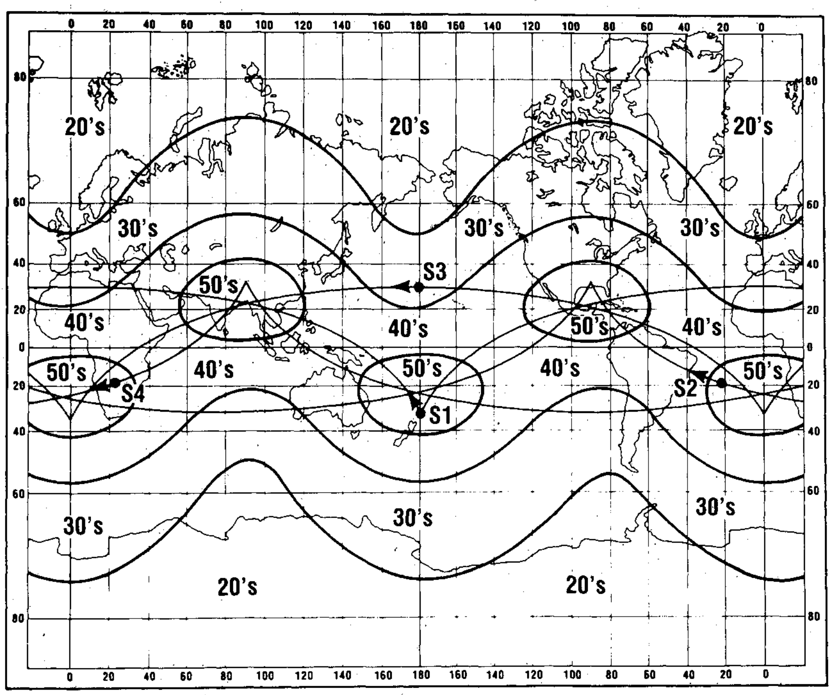

Fig. 14 Example constellation mean look-angle ranges by geographic area.

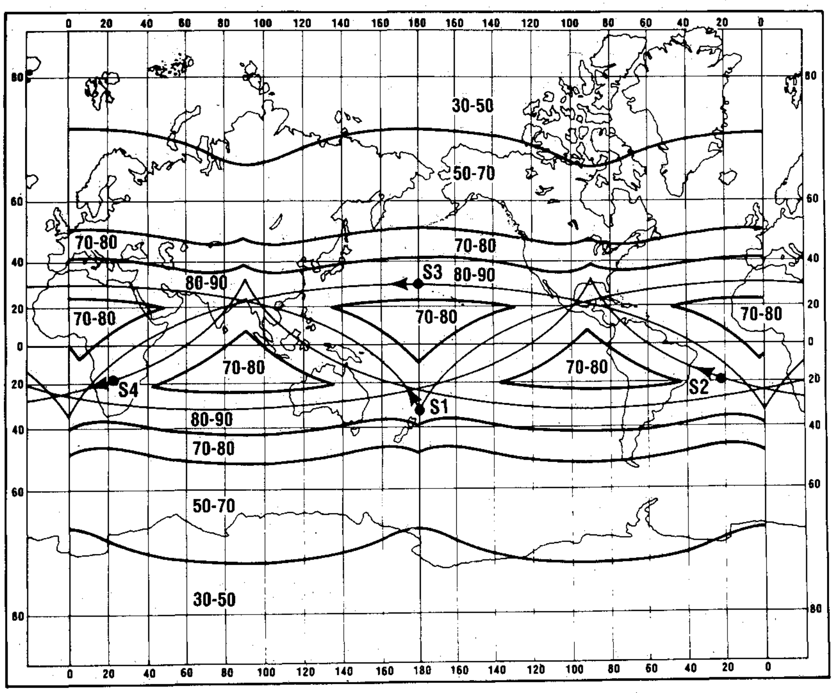

Fig. 15 Example constellation maximum look-angle ranges by geographic area.

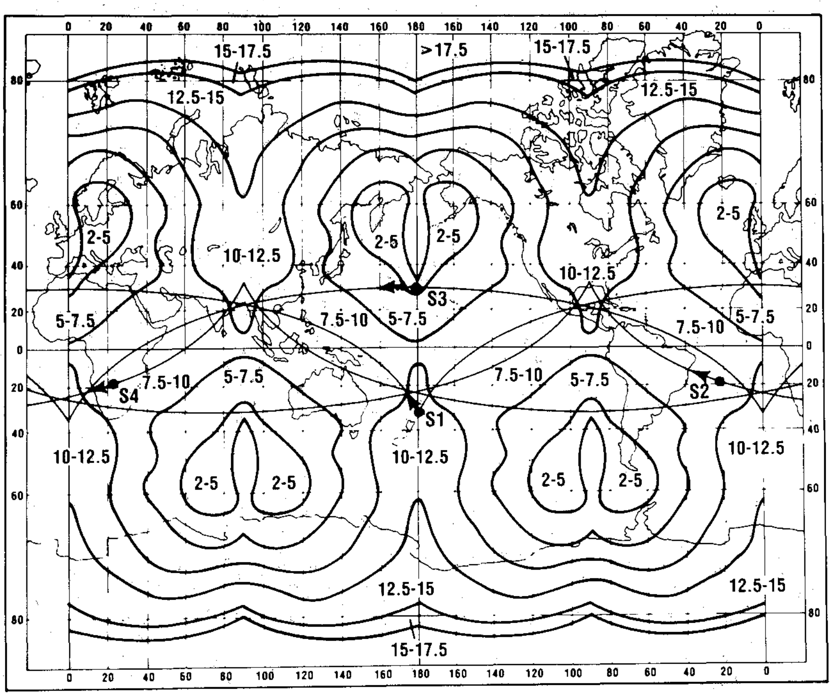

Fig. 16 Example constellation minimum look-angle ranges by geographic area.

Estimates for the rotation of perigee and regression of right ascension of the line of nodes were made using Eqs. (3) and (6) from Ref. 12 for the 48 h example constellation described in this paper.

The regression of nodes due to Earth oblateness is,[12]

$$\dot{\Omega} = -\frac{3}{2a}\sqrt{\frac{\mu}{a}}\, J_2 \left(\frac{R_e}{p}\right)^2 \cos i \tag{5}$$

This regression rate, evaluated for the 48 h example constellation, is about 0.955 deg/yr.

Since $\dot{\omega}$ is the only one of the above two perturbations to affect the coverage integrity of the constellation, it is the only one that will require an investment in maneuvering or stationkeeping.

The rotation of perigee due to Earth oblateness is to a first order,[12]

$$\dot{\omega} = \frac{3}{4a}\sqrt{\frac{\mu}{a}}\, J_2 \left(\frac{R}{p}\right)^2 \left(-1 + 5\cos^2 i\right) \tag{6}$$

Again, using the parameter values for the 48 h example constellation, the rotation of perigee is 0.177 deg/yr. At these high altitudes, the maneuvering requirement to counter this perturbation is quite small (ΔV less than 20 ft/s per year). Since this rate is so low, it is evident that only 1 or 2 deg of

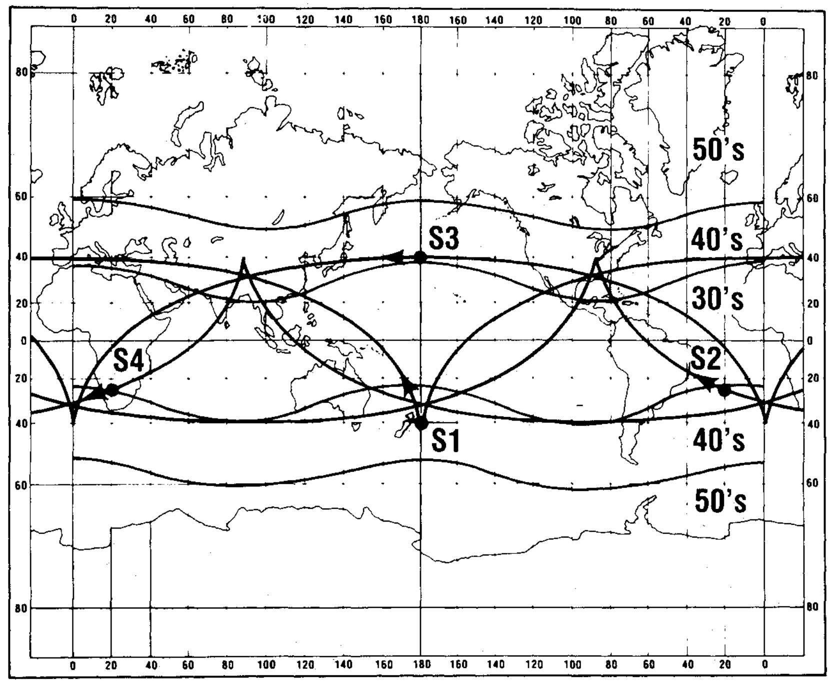

Fig. 17 Modified constellation mean look-angle ranges by geographic area ($T = 48$ h, $i = 40$ deg, $e = 0.34$).

perigee rotation should be expected in the average satellite lifetime (5–10 yr). Test cases with the 48 h example constellation indicate that up to 5 deg of perigee rotation is acceptable (i.e., the constellation will maintain continuous visibility of the Earth's surface). To hedge against rotation of perigee, another possibility is to put an initial bias $\Delta\omega$ equal to one-half the expected satellite lifetime perigee shift. Then, the midterm ω will be ± 90 deg (the optimum value) with the maximum perigee excursions at the beginning and end of the satellite lifetime.

Conclusion

A common-period four-satellite, elliptic orbit constellation can provide continuous line-of-sight coverage of the entire Earth's surface for periods of 26.49 h or greater. Two of the satellites in the constellation have their perigees in the Northern Hemisphere, while the other two have their perigees in the Southern Hemisphere. For periods greater than 26.49 h, a range of inclination and eccentricity values may be used. The inclination angle was found to be 31.3 deg and the eccentricity 0.263 to minimize the constellation period while maintaining continuous visibility.

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